Observability singularity of batch bioreactors: a solution based on high order sliding mode differentiator approach.

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February 9, 2018

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- Some recalls on observability singularities
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- Reconstruction of the substrate
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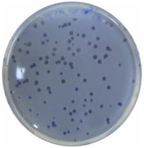
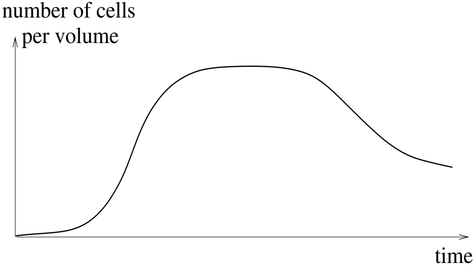
Introduction

Bacterial growth in batch





e.g. *J. Pirt. Principles of microbe and cell cultivation*, Wiley 1975

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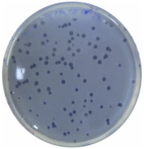

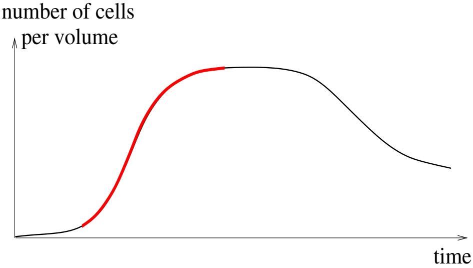

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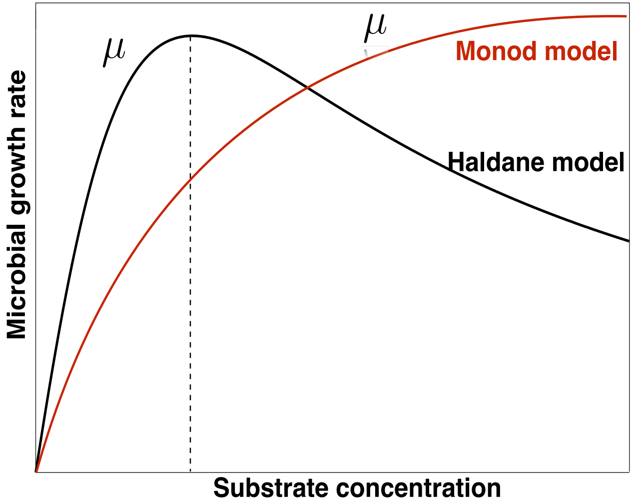
Introduction



Introduction

The batch culture is characterized by the fact that after the initial charge of the substrate in the bioreactor and biomass inoculation, there is no inflow or outflow of the medium. A typical model is

$$\begin{aligned} \dot{b} &= \mu(s)b \\ \dot{s} &= -\mu(s)b \end{aligned} \tag{1}$$

where s is the substrate concentration, b is the biomass concentration and $\mu(\cdot)$ is the microbial growth rate function.

Introduction

Several models of microbial growth have been discussed in the literature. The Haldane's model is a popular one that describes the dynamics of the growth of a biomass which is inhibited by high substrate concentration. This is given by the following

$$\mu(s) = \frac{\bar{\mu} s}{K_s + s + \frac{s^2}{K_i}} \qquad (2)$$

where $\bar{\mu}$, K_s and K_i are positive parameters.

Introduction

Goal: reconstruct the substrate concentration from (1) when only the biomass b is measured.

Problem: The observability matrix at the order 2 of system (1), given by

$$dO_2 = \begin{pmatrix} 1 & 0 \\ \mu(s) & \frac{\partial \mu(s)}{\partial s} b \end{pmatrix}, \tag{3}$$

is singular at $s = \sqrt{K_i K_s}$.

Here, we overcame the observability problem of system (1)–(2). Using a second order sliding mode differentiator we construct a convergent observer allowing to compute (in finite time) $\dot{y}$ and $\ddot{y}$, the first and second derivative of the output y.

Knowing that it is a finite time convergence, the comparison of $\dot{y}$ and $\ddot{y}$ with the original system permits to build a test procedure allowing the exact construction of the substrate concentration by considering the original parameters system known.

Problem Statement

Note that if $s(0)$ and $b(0)$ are positives then the system is positive. Then, from the following change of coordinates

$$(y, s)^T = (\log(b), s)^T$$

the system (1) becomes

$$\begin{aligned} \dot{y} &= \mu(s) \\ \dot{s} &= -\mu(s)e^y \end{aligned} \quad (4)$$

and the output

$$y = log(b).$$

Problem Statement

The observability matrix at the order 2 of system (4), given by

$$dO_2 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{\partial \mu(s)}{\partial s} \end{pmatrix}, \tag{5}$$

with

$$\frac{\partial \mu(s)}{\partial s} = \frac{\bar{\mu}}{K_i} \frac{K_s K_i - s^2}{\left(K_s + s + \frac{s^2}{K_i}\right)^2} \tag{6}$$

shows that the system has a observability singularity set

$$SO_2 = \{(b, s) \setminus s = \sqrt{K_i K_s}\}. \tag{7}$$

Note that the solution $s = -\sqrt{K_i K_s}$ is not a possible solution with respect to a biological purpose.

Problem Statement

By considering the second derivative of the output, the observability matrix in this case is given by

$$dO_3 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{\partial \mu(s)}{\partial s} \\ \frac{\partial \ddot{y}}{\partial y} & \frac{\partial \ddot{y}}{\partial s} \end{pmatrix} \tag{8}$$

with

$$\frac{\partial \ddot{y}}{\partial s} = -e^{y} \left[(\frac{\partial \mu}{\partial s})^2 + \frac{\partial^2 \mu(s)}{\partial s^2} \mu(s) \right] \tag{9}$$

Problem Statement

$$\frac{\partial^2 \mu(.)}{\partial s^2} = \frac{-2\bar{\mu}(K_s K_i + 4K_s s + s^2)}{K_i(K_s + s + \frac{s^2}{K_i})^3}. \qquad (10)$$

Then, when $s = \sqrt{K_s K_i}$ (i.e. $\frac{\partial \dot{y}}{\partial s} = 0$), $\frac{\partial \ddot{y}}{\partial s}$ can be also equal to zero only if $b = 0$ (i.e. $y = log(b) = -\infty$). This solution is impossible in finite time with respect to the model. Consequently, we conclude that considering dO_3, the set of singularity is:

$$SO_3 = \emptyset.$$

Introduction

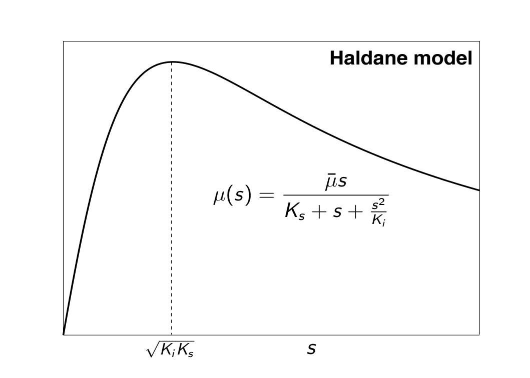


Figure: The model of Haldane.

High order sliding mode observer

The following high order sliding mode differentiator is considered in order to compute y, $\dot{y}$, $\ddot{y}$ (see A. Levant)

$$\begin{aligned} \dot{z}_1 &= z_2 - K_1 L^{\frac{1}{3}} |z_1 - y|^{\frac{2}{3}} sign(z_1 - y) \\ \dot{z}_2 &= z_3 - K_2 L^{\frac{1}{2}} |z_1 - y|^{\frac{1}{3}} sign(z_1 - y) \\ \dot{z}_3 &= -K_3 L sign(z_1 - y) \end{aligned} \tag{11}$$

The vector (z_1, z_2, z_3) converges in finite time to $(y, \dot{y}, \ddot{y})$.

Reconstruction of the substrate

Considering that the finite time observer (11) has converged, equation (2) and $z_2 = \mu(s)$ gives the following quadratic equation:

$$z_2 s^2 + (z_2 K_i - \bar{\mu} K_i) s + z_2 K_s K_i = 0 \qquad (12)$$

which has two real solutions

$$\begin{cases} S_1 = \dfrac{-(z_2 K_i - \bar{\mu} K_i) + \sqrt{(z_2 K_i - \bar{\mu} K_i)^2 - 4 z_2^2 K_s K_i}}{2 z_2}, \\ S_2 = \dfrac{-(z_2 K_i - \bar{\mu} K_i) - \sqrt{(z_2 K_i - \bar{\mu} K_i)^2 - 4 z_2^2 K_s K_i}}{2 z_2}. \end{cases}$$

The singularity in $\dot{y} = 0$ for S_1 and S_2 is not a real singularity because this correspond to $\mu(s) = 0$ and then $s = 0$, so in this case s is known.

Reconstruction of the substrate

Now in order to determine between S_1 and S_2 which is the solution of (4), it is enough to use the information given by $\ddot{y}$. For that, using the fact that after a finite time $\ddot{y} = z_3$, it is enough to determine if it is S_1 or S_2 which verifies the following equality:

$$\ddot{y} = z_3 = -\frac{\partial \mu}{\partial s} \mu e^y.$$

or again

$$\ddot{y} = z_3 = -\bar{\mu}^2 \left(K_s - \frac{s^2}{K_i} \right) \frac{e^y s}{(K_s + s + \frac{s^2}{K_i})^3}. \qquad (13)$$

Reconstruction of the substrate

This gives the following test procedure, defining the function *Test*

$$Test = z_3 + \bar{\mu}^2 (K_s - \frac{s^2}{K_i}) \frac{s e^y}{(K_s + s + \frac{s^2}{K_i})^3} \qquad (14)$$

and we chose the solution S_1 if for $s = S_1$ the function $Test = 0$ (or if it is the closest to zero) and we chose the solution S_2 if for $s = S_2$ the function $Test = 0$ (or if it is the closest to zero).

Simulation results

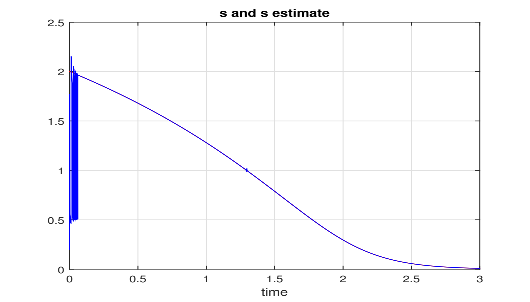


Figure: The substrat s in red, and its estimate in blue.

Simulation results

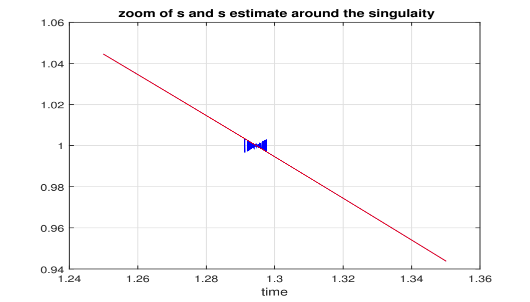


Figure: Zoom around the singularity of the substrat *s* in red, and its estimate in blue.

Conclusion

This paper has highlighted, first the efficiency of the High Order Sliding mode observer and second the possibility to overcome an observability singularity of batch bioreactors by the mean of a simple algebraic test thanks to the finite time convergence of the derivatives of the measurement signal. Our future works will be devoted to taking into account the fact that the biomass measurement is sampled. The confrontation of our observer with experimental data of y, $\dot{y}$ and $\ddot{y}$ is one of the main ends of future works.